MATH 116 – Spring 2005
Practice Problems for Exam 1

Problem 1. Circle the correct answer.
(I) $\ln(\ln(e^x))$ is equal to
$(A)\ \ln x \quad (B)\ e^x \quad (C)\ \ln e \quad (D)\ e \quad (E)$ None of the above.
(II)$\sqrt{9e^{6x}}$ is equal to
$(A)\ 3e^{\sqrt{6x}} \quad (B)\ 3+e^{3x} \quad (C)\ 3e^{3x} \quad (D)\ 3(\sqrt{e})^{6x} \quad (E)$ None of the above.
(III) $\ln(\dfrac{x}{4})+\ln 4$ is equal to
$(A)\ \ln(\frac{x}{4}+4) \quad (B)\ \ln x \quad (C)\ 4\ln x \quad (D)\ \ln(\frac{x}{4})\ln 4 \quad (E)$ None of the above.
(IV) The inverse of the function $f(x)=\ln(x)$ is
$(A)\ g(x)=(\ln x)^{-1} \quad (B)\ g(x)=e^x (C)\ g(x)=e^{-x} \quad (D)\ g(x)=\ln(\frac{1}{x}) \quad (E)$ None of the above.

Problem 2. Find the derivative of the following functions
(A) $f(x)=\ln(x^3+1)$.
(B) $f(x)=e^{x^2-\sqrt{x}}$.
(C) $f(x)=e^x\ln(e^x+1)$.
(D) $f(x)=\cos(\mathrm{e}^x)$.
(E) $f(x)=\ln(\sin(x^2))$.
(F) $f(x)=\sin(\sin(\sqrt{x}))$.

Problem 3. Find the maximum and minimum values of the following function on the given intervals. Justify your work.
(A) $f(x)=2+\sin(x), \quad x\in[0,2\pi]$.
(B) $f(x)=\sqrt{\sin x+2}, \quad x\in[0,2\pi]$.
(C) $f(x)=x-\sin(x), \quad x\in[\pi/2,3\pi/2]$.
(D) $f(x)=6(1-\sin\frac{\pi x}{2}) \quad x\in[1/2,5/4]$.

Problem 4.
Find the coordinates of all relative extrema and inflection points of $f(x)=\dfrac{x^2}{e^x}$.

Problem 5. Compute the following integrals. Check your answers.
(A) $\int 2\sin(2x)\,dx$.
(B) $\int x(x^2+2x\pi)\,dx$.
(C) $\int e^{4x}\,dx$.
(D) $\int (3x^{3/2}+\dfrac{1}{x})\,dx$.
(E) $\int (4x-12x^3)(2x^2-3x^4)^{17}\,dx$.

Problem 6. An architect wishes to design a ramp inclined 30^0 leading from the ground level to a second-story door in a parking garage. How far from the building must the ramp start if the garage is 12 feet above the ground level?

Problem 7. A patient's temperature is 108 degrees and is changing at the rate of t^2-4t degrees per hour, where t is the number of hours since taking fever-reducing medication. Find the patient's temperature after 2 hours.

Brief Solutions

(you may have to show more work in the exam)

Problem 1.
(I) $\ln(\ln(e^x)) = \ln x$ **(II)** $\sqrt{9e^{6x}} = 3e^{3x}$, **(III)** $\ln(\dfrac{x}{4}) + \ln 4 = \ln x$, **(IV)** The inverse of the function $f(x) = \ln(x)$ is $g(x) = e^x$.

Problem 2.
(A) $f'(x) = \dfrac{3x^2}{x^3+1}$.
(B) $f'(x) = e^{x^2-\sqrt{x}}(2x - \dfrac{1}{2\sqrt{x}})$.
(C) $f'(x) = e^x \ln(e^x+1) + \dfrac{e^{2x}}{e^x+1}$.
(D) $f'(x) = -\sin(e^x)\, e^x$.
(E) $f'(x) = \dfrac{1}{\sin(x^2)}\ \cos(x^2)\, 2x = 2x\cot(x^2)$.
(F) $f'(x) = \cos(\sin(\sqrt{x}))\ \cos(\sqrt{x})\ \dfrac{1}{2\sqrt{x}}$.

Problem 3.
(A) $f(3\pi/2) = 1$ and $f(\pi/2) = 3$ (f is $\sin x$ shifted two units up).
(B) $f'(x) = \dfrac{\cos x}{2\sqrt{\sin x + 2}}$, $f'(x) = 0$ for $x = \pi/2$ or $x = 3\pi/2$. Max=$f(\pi/2) = \sqrt{3}$, Min=$f(3\pi/2)$=1.
(C) $f'(x) = 1 - \cos x$, $f'(x) \neq 0$ on $[\pi/2, 3\pi/2]$, Max=$f(3\pi/2) = 3\pi/2 + 1$, Min=$f(\pi/2) = \pi/2 - 1$
(D) $\sin(x\pi/2)$ is positive on [1/2,5/4] and $\sin(x\pi/2) = 1$ there for $x = 1$. Max = $f(1/2) = 6(1 - \frac{\sqrt{2}}{2})$, Min = $f(1) = 0$.

Problem 4.
$f'(x) = 2xe^{-x} - x^2e^{-x} = e^{-x}(2x - x^2)$
$f''(x) = -e^{-x}(2x - x^2) + e^{-x}(2 - 2x) = e^{-x}(x^2 - 4x + 2)$
$f'(x) = 0 \quad \leftrightarrow \quad e^{-x}(2x - x^2) = 0 \quad \leftrightarrow (2x - x^2) = 0 \quad \leftrightarrow x = 0 \ \text{ or } \ x = 2.$
$f''(0) = 2 > 0$, then $(0,0)$ **is a local minimum**.
$f''(2) = -2e^{-2} < 0$, then $(2, 4e^{-2})$ **is a local maximum**.
$f''(x) = 0 \leftrightarrow \quad e^{-x}(x^2 - 4x + 2) = 0 \leftrightarrow \quad (x^2 - 4x + 2) = 0 \leftrightarrow x = 2 \pm \sqrt{2}$
$f''(0) > 0$ and $f''(2) < 0$, then $(2 - \sqrt{2}, f(2 - \sqrt{2}))$ **is an inflection point.**
$f''(2) < 0$ and $f''(4) > 0$, then $(2 + \sqrt{2}, f(2 + \sqrt{2}))$ **is an inflection point.**

Problem 5.
(A) $-\cos 2x + C$.
(B) $1/4x^4 + 2/3x^3\pi + C$.
(C) $\dfrac{1}{4}e^{4x} + C$.
(D) $(6/5)x^{5/2} + \ln|x| + C$.
(E) $(1/18)(2x^2 - 3x^4)^{18} + C$

Problem 6.
$30^0 = \pi/6$, $1/\sqrt{3} = \tan(\pi/6) = 12/x$ and hence $x = 12\sqrt{3}$ feet.

Problem 7.
$T(t) = 1/3t^3 - 2t^2 + C$.
$T(0) = 108$ so $C = 108$, $T(2) = 1/32^3 - 2\,2^2 + 108 = 102.67$ degrees.