MATH 116 – PRACTICE PROBLEMS FOR THE FINAL EXAM

Problem 1. Circle the correct answer.
(I) The area between the graphs of $y = \cos x$ and $y = \sin x$ for $0 \le x \le \pi/4$ is given by

$$(A)\ \int_0^{\pi/4} (\sin x - \cos x) dx \quad (B)\ \int_0^{\pi/4} (\cos x - \sin x) dx \quad (C)\ \int_0^{\pi/4} (\sin x + \cos x) dx$$

$$(D)\ \int_0^{\pi/4} (1 - \sin x + \cos x) dx \quad (E) \text{ None of the previous.}$$

(II) The integral $\int g(x) f'(x)\, dx$ is equal to

$$(A)\ \int g'(x) f(x)\, dx \quad (B)\ g(x)f(x) + C \quad (C)\ g(x)f(x) - \int g'(x)f(x)\, dx$$

$$(D)\ g'(x)f(x) + g(x)f'(x) \quad (E) \text{ None of the previous.}$$

(III) The trapezoidal rule for definite integrals with $n = 4$ gives $\int_0^1 4\, dx =$

$$(A)\ 1/5(0/2 + 0.2 + 0.4 + 0.8 + 1/2) \quad (B)\ 3.95 \quad (C)\ 1/5(0/2 + 2 + 4 + 8 + 10/2)$$

$$(D)\ 4 \quad (E) \text{ None of the previous.}$$

(IV) $\int_1^{\infty} \mathrm{e}^x\, dx =$

$$(A)\ \infty \qquad (B)\ \mathrm{e} \qquad (C)\ 0 \qquad (D)\ 1 \qquad (E) \text{ None of the previous.}$$

(V) $\sum_{k=0}^{99} 2^k =$

$$(A)\ 2^{99} \quad (B)\ 200/(200-1) \quad (C)\ 20,0000,000,000,000 \quad (D)\ 2^{100} - 1$$

(E) None of the previous.
(VI) The function $f(x) = \sin(\pi x)$ has a local maximum at

$$(A)\ x = 0 \quad (B)\ x = -1 \quad (C)\ x = 1/2 \quad (D)\ x = -1/2 \quad (E) \text{ None of the previous.}$$

(VII) Assume that $f(1,2) = 6, f_x(1,2) = 2$, and $f_y(1,2) = -1$. Then, the approximate value of $f(1.1, 2.1)$ obtained using the given information and differentials is

$$(A)\ 0 \quad (B)\ 6.1 \quad (C)\ 6 \quad (D)\ 5.9 \quad (E) \text{ None of the previous.}$$

(VIII) The sum of the infinite geometric series

$$4 + \frac{4}{5} + \frac{4}{25} + \frac{4}{125} + ...$$

is the number

$$(A)\ \infty \qquad (B) \frac{16}{5} \qquad (C)\ 5 \qquad (D)\ (\frac{1}{5})^n \qquad (E) \text{ None of the previous.}$$

Problem 2. Compute the indicated derivative or partial derivatives.
(A) $f(x) = \mathrm{e}^{\cos x}$. $\quad f'(\pi/2) =$
(B) $f(x, y) = y^2 \sin\sqrt{x^2+1}$. $\quad \dfrac{\partial^2 f}{\partial y^2}(x, y) =$
(C) $f(x, y) = x \sin y + y\mathrm{e}^x + \sin(xy) + x^2$. $\quad \dfrac{\partial f}{\partial y}(x, y) =$

Problem 3. Compute the following integrals.

(A) $\displaystyle\int x\mathrm{e}^{2x}\, dx$

(B) $\displaystyle\int x^{3/2} \ln x\, dx$

(C) $\displaystyle\int \frac{\cos(\ln x)}{x}\, dx$

Problem 4.
(A) Find the critical point or points of $f(x, y) = x^2 + y^3 - 6x - 12y$.
(B) Compute all second derivatives of $f(x, y)$.
(C) Using the second derivative test, classify the critical point or points from (A) as relative maximum, relative minimum, or saddle point.

Problem 5. Compute the volume under the graph of the function $f(x, y) = xy + 3x^2$ over the region $0 \le x \le 1$ and $0 \le y \le 2$.

Problem 6. A chemical refinery discharges $P(t) = 5 + 3\sin(\frac{\pi t}{6})$ tons of pollution each month, where t is the number of months that the plant has been in operation. Find the average amount of pollutant discharged for the first eighteen monts($t = 0$ to $t = 18$).

Problem 7. In a psychology experiment rats were placed in a maze. It was found that the proportion of rats who required more than t seconds to leave the maze was given by

$$P(t) = \int_t^{\infty} 0.05e^{-0.05x} dx.$$

Compute the proportion of the rats that required more than 10 seconds to leave the maze.

Problem 8. The function $f(x, y) = 6x^2 - y^2 + 4$ subject to the constraint $3x + y - 12 = 0$ has only one maximum. Compute the coordinates (x, y) of the point where f attains its maximum and the value of such maximum.

Problem 9. A company makes two kinds of umbrellas A and B. Type A sells for $\$8$ each, while type B sells for $\$12$ each. The cost of making x umbrellas of type A and y umbrellas of type B is given by $C(x, y) = x^2 - xy + y^2 - 4x + 6y + 2$ (in dollars). How many umbrellas of each kind should be made in order to maximize profit and what is the maximum profit? (Make sure you show work that justifies that your answer is really a maximum.)

Problem 10. Compute the double integral of the function $f(x, y) = xy + x^2y$ over the region $R = \{1 \le x \le 3,\ x \le y \le x^2\}$.

Problem 11. Do again the parctice problems for the midterms.