Math 123 – Calculus III
Practice Problems – Midterm Exam
Spring 2004

The midterm exam is on **Tuesday, March 16, 5:45p.m.–7:45p.m. – 1005 Haworth.**

The exam will cover the following sections from the textbook:

- Sections B1.5, B1.6, B1.7, B2.1, B2.2, B2.3, B2.4, B3.1, B3.2, B.3.3, B3.4, B3.5, B3.6.
- Sections S1.3, S1.4.

These problems are intended to be used as part of a review for the midterm exam. Do not confuse this, however, with a sample midterm. The problems below are not a substitute for studying all the material covered in class and the homework assignments. Review also the material for the first test.

Problems from the text book:

- Sec. B2.3: 2, 14c), 17a).
- Sec. B2.4: 1, 8.
- Sec. B3.1: 3, 12.
- Sec. B3.2: 13, 25.
- Sec. B3.3: 2, 10.
- Sec. B3.4: 8, 10, 13, 17, 20, 27.
- Sec. B3.5: 4, 8, 17, 19.
- Sec. B3.6: 1, 7, 13, 15, 19, 23, 24, 27.

Additional problems:

1. Answer the following true or false.

   a) T F Let A, B be $n \times n$ invertible matrices. Then $(AB)^{-1} = A^{-1}B^{-1}$.

   b) T F A homogeneous system of three equations in four unknowns has no solution.

   c) T F Let $A\mathbf{x} = \mathbf{b}$ be an $m \times n$ linear system. Then $\mathbf{b} \in \mathbf{R}^n$.

2. Given $g(x, y) = \sqrt{x^2 + y^2}$, (a) Sketch the level curves of this function in the xy-plane, (b) sketch the graph of the function.

3. If $g, h : \mathbf{R}^2 \to \mathbf{R}$ and $f : \mathbf{R}^3 \to \mathbf{R}$ are functions with continuous partial derivatives and $F : \mathbf{R}^3 \to \mathbf{R}$ is defined by $F(s, t, u) = f\left(g(s, t), tu^2, h(s, u)\right)$. Express $\partial F/\partial u$ in terms of the partial derivatives of the functions $f(x, y, z)$, $g(s, t)$ and $h(s, u)$.

4. Find (a) the Jacobian matrix of $f(x, y) = (x^2y - e^y, xe^{xy}, e^x y^3)$ at $(1, -1)$ and (b) the Jacobian matrix of $g(s, t, u) = (s^3tu^2 - su, st, t^u)$ at $(-1, 0, -1)$.

5. Given $f : \mathbf{R}^2 \to \mathbf{R}^2$ and $g : \mathbf{R}^2 \to \mathbf{R}^2$ such that $f(1, 1) = (1, 0)$, $J(g \circ f)(1, 1) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $Jg(1, 0) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$, find $Jf(1, 1)$.

6. The augmented matrix of a sytem of equations and its reduced row echelon form is given below.

$$\begin{bmatrix} 1 & 2 & 0 & 4 & -5 \\ -3 & -6 & 1 & -9 & 22 \\ 4 & 8 & 0 & 16 & -19 \end{bmatrix} \longrightarrow rref \begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   a) Does the system have solutions?

   b) Find all the solutions of the linear system

$$\begin{bmatrix} 1 & 2 & 0 \\ -3 & -6 & 1 \\ 4 & 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -9 \\ 16 \end{bmatrix}.$$

7. For what values of a is $A = \begin{bmatrix} a-2 & 2 \\ a-2 & a+2 \end{bmatrix}$ singular?

   (A) 0, 2 (B) -2, 4 (C) 2, 4 (D) 2 (E) 0, -2

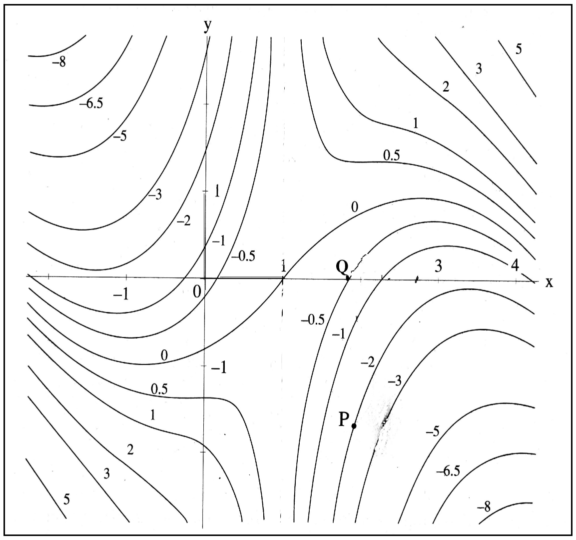
8. The matrix expression $(A + B)(A - B)$ can be rewritten as
   (a) $A^2 - B^2$ (b) $A^2 - AB + BA - B^2$ (c) $A^2 - BA + AB - B^2$ (d) all of the above (e) none of the above

9. Which of the following is a reduced row echelon form?

   (A) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

   (E) none of the above

10. The figure below represents level sets of a function $f : \mathbf{R}^2 \to \mathbf{R}$.



(a) At the point P, is $\dfrac{\partial f}{\partial y}$ positive or negative?

(b) Estimate $\dfrac{\partial f}{\partial x}$ at the point $Q = (2, 0)$.