MATH 124 – Fall 2004
Practice Problems for Exam 3

1. (a) Compute the line integral of the vector field $\mathbf{F} = (3x^2y, x^3 + 3y^2)$ along the segment from $(1,1)$ to $(2,2)$ by direct computation.
   (b) Show that $\mathbf{F} = (3x^2y, x^3 + 3y^2)$ is a conservative vector field.
   (c) Find a potential for $\mathbf{F} = (3x^2y, x^3 + 3y^2)$.
   (d) Compute the line integral of the vector field $\mathbf{F} = (3x^2y, x^3 + 3y^2)$ along any curve from $(1,1)$ to $(2,2)$.

2. (a) Consider the vector field $\mathbf{G}(x,y) = (2y + x^3, x)$. Show that $\mathbf{G}$ is not conservative.
   (b) Compute the line integral
   $$\oint_C \mathbf{G} \cdot d\mathbf{x},$$
   where the curve C is the boundary of the square $[0,1]\times[0,1]$ oriented counterclockwise.

3. Consider the curve γ given by the three sides of the triangle from $(0,0)$ to $(1,0)$ to $(0,1)$ oriented counterclockwise. Show that
   $$\int_\gamma (-xy + \sin x^2)\, dx \ + \ \cos y^2\, dy = 1/6.$$

4. Show that the line integral of $\mathbf{F} = (x/(x^2+y^2), y/(x^2+y^2))$ along any closed curve that does not have the point $(0,0)$ in its interior is zero.

5. Let $\mathbf{F}$ and $\mathbf{G}$ are vector fields in R^3 (appropriately differentiable). Show that
   $$\mathrm{div}(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\nabla\times\mathbf{F} - \mathbf{F}\cdot\nabla\times\mathbf{G}.$$

6. Compute the flux of the vector field $\mathbf{F}(x,y,z) = (x,y,z)$ across the sphere of radius one in the direction to the normal pointing to the inside.

7. Let S be the parametric surface given by
   $$\mathbf{X}(x,z) = (x, x^3 + z, z),$$
   for $0 \le x \le 2$ and $0 \le z \le 3$.
   (a) Find the equation of the tangent plane to surface S at the point (1,2,1).
   (b) Set up an integral to compute the area of the parametric surface S. DO NOT COMPUTE THE INTEGRAL.

8. (a) Compute the unit normal vector $\mathbf{n}$ pointing to the outside at each point of the cylinder given by
   $$\mathbf{X}(u,v) = (\cos u, \sin u, v)$$
   for $0 \le u \le 2\pi$, $0 \le v \le 1$.

(b) Compute the flux of the vector field $\mathbf{F}$ across the cylinder S of part (a),

$$\text{FLUX } = \int\int_S \mathbf{F} \cdot d\mathbf{S},$$

where

$$\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

9. Compute

$$\int\int_S z^3 dS$$

where S is the sphere of radius one centered at the origin.

10. Consider the paraboloid M given by $z = 1 - (x^2 + y^2)$ for $0 \leq x^2 + y^2 \leq 1$.

(a) Write a parametrization of M of the form $\mathbf{f}(x, y) = (x, y, h(x, y))$.

(b) Compute the unit normal to the surface M, ”pointing up”.