MATH 124 – Fall 2004
Practice Problems for Exam 1

1. A point has position at time t given by $\mathbf{x}(t) = (t, t^2, 1 + t^2)$, for $0 \le t \le 1$. At time $t = 1$ the point leaves this curve and flies off along the tangent line while maintaining the constant velocity attained at $t = 1$. Where is the point at at $t = 2$?

2. Consider the surface given by the graph of the function $f(x, y) = 2y^2 - x^2 y$.
   a) Find an equation of the tangent plane to the surface at the point $P(1, 1, 1)$.
   b) Find the equation of the normal line to the surface at the same point.
   c) A curve given by the vector valued function $\mathbf{x}(t)$ satisfies that $\mathbf{x}(1) = (1, 1, 1)$ and $\mathbf{x}'(1) = (4, 0, 1)$. Can a small arc of the curve be contained in the given surface for t close to 1? Justify your answer.

3. A cyclist goes on a mountain road and, at time t, her position path is given by $\mathbf{x}(t) = (t, 2/3t^{3/2}, 2 - t^2)$. On which of the following two mountains does her path lie?

   Mountain 1 has height $z = H_1(x, y) = 2 + 9/4y^2 - x^3 - x^2$.

   Mountain 2 has height $z = H_2(x, y) = 2 + 3/2y - x^2 - x^3$.

4. A particle is traveling in space with its position given by $\mathbf{x}(t) = t^2\,\vec{\imath} + t\,\vec{\jmath} + 2t^2\,\vec{k}$.
   (a) Find the velocity vector of the particle at time $t = 4$.
   (b) Find the acceleration vector for the particle as a function of t.

5. A caterpillar is standing on a hill whose height is given by

$$H(x, y) = x^3 - xy$$

   Take "North" to be the direction of the positive y-axis and "East" to be the positive x-axis and use this information to answer the following questions:

   (a) If the caterpillar is standing at the point $(2, 1)$ and begins walking South, is it going uphill, downhill, or neither?

   (b) If the caterpillar is standing at the point $(1, 1)$ and walks 1 units South, then 2 units West, what is the total change in elevation?

   (c) If the caterpillar is standing at the point $(-1, 2)$, describe the direction (by a unit vector) that the caterpillar should move to go uphill fastest.

6. Circle only one of the given answers for each question.

   a) Let $f(x, y, z) = e^{x+y}\cos z$. Then $\nabla f(0, 0, \pi)$ is

   (A) $-\vec{\imath} - \vec{\jmath}$ (B) $\vec{\imath} + \vec{\jmath}$ (C) $\vec{\imath} - \vec{\jmath} + \vec{k}$ (D) $\vec{\imath} + \vec{\jmath} - \vec{k}$

   b) The tangent plane to the surface $x^4 - xy + z^2 = 1$ at $(0, 1, 1)$ is
   (A) $-x + 2z = 2$ (B) $y + z = 2$ (C) $-x + y + 2z = 3$ (D) $2z = 2$

c) The level surfaces of the function $f(x, y, z) = x + 2y + z - 5$ are
(A) planes perpendicular to $(1, 2, 1)$ (B) planes perpendicular to $(1, 2, -5)$
(C) concentric spheres (D) none of the previous.

7. Suppose that the gradient $\nabla f(2, 4)$ of a function $f(x, y)$ has length 5. Is there a unit vector $\mathbf{u}$ for which the directional derivative $D_{\mathbf{u}} f$ at the point $(2, 4)$ is 7? Justify your answer.

8. Calculate the area of the triangle with vertices (1,2,3), (4,-2,1) and (-3,1,0).

9. Give a set of parametric equations for the plane determined by $2x + 3y - 5z = 30$.

10. Determine all second order partial derivatives of the function $f(x, y) = \sin \sqrt{x^2 + y^2}$.

11. Compute the matrix of partial derivatives $D\mathbf{f}(\mathbf{a})$ where $\mathbf{f}(s, t) = (s^2, st, t^2)$ and $\mathbf{a} = (1, -1)$

12. Compute the matrix of partial derivatives $D(\mathbf{f} \circ \mathbf{g})$, where $\mathbf{f}(x, y, z) = (x+y+z, xyz, e^z)$ and $\mathbf{g}(s, t) = (st, s + t, t^3)$.

13. Let $\mathbf{f} : \mathbf{R}^2 \to \mathbf{R}^2$ and $\mathbf{g} : \mathbf{R}^2 \to \mathbf{R}^2$ be two functions such that $\mathbf{f}(1, 1) = (1, 0)$, $D\mathbf{f}(1, 1) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$, $D\mathbf{f}(1, 0) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $D\mathbf{g}(1, 1) = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, and $D\mathbf{g}(1, 0) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Then,

$$D(\mathbf{g} \circ \mathbf{f})(1, 1) =$$

(A) $\begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$

(E) None of the above.

14. Mark all the correct answers. Let $f(x, y)$ be a function of two variables. Then, $\frac{\partial f}{\partial x}(x_0, y_0)$ is equal to

(A) $\frac{\partial f}{\partial y}(x_0, y_0)$.

(B) the rate of change of f with respect to x at the point (x_0, y_0) when $y = y_0$ is fixed.

(C) the slope of the tangent line to the curve of intersection of the plane $y = y_0$ with the graph of $z = f(x, y)$ at the point (x_0, y_0).

(D) all of the above.

15. Write an equation for the tangent plane to the surface given by the level set corresponding to the value 3 of the function $F : R^3 \to R$, $F(x, y, z) = y^4 x + z^2 y + zx^3$, at the point $(1, 1, 1)$.