MATH 124 – Fall 2004
Practice Problems for Exam 2

1. Fill in the blanks with ONE of the following options HAS, MAY HAVE, DOES NOT HAVE. Let $f(x,y)$ be a function with continuous second order partial derivatives and let $Hf(x_0, y_0)$ be the Hessian matrix evaluated at (x_0, y_0).

   (a) If $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$, $f_{xx}(x_0, y_0) = 0$, and $f_{xy}(x_0, y_0) < 0$, then f ______________ a saddle point at (x_0, y_0).

   (b) If $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$, $f_{xx}(x_0, y_0) < 0$, and $f_{yy}(x_0, y_0) < 0$, then f ______________ a local maximum at (x_0, y_0).

   (c) If $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$, det$Hf(x_0, y_0) > 0$, and $f_{xx} > 0$, then f ______________ a local minimum at (x_0, y_0).

   (d) If det$Hf(x_0, y_0) = 0$, $f_x(x_0, y_0) > 0$, and $f_y(x_0, y_0) > 0$, then f ______________ a local maximum at (x_0, y_0).

   (e) If $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$, det$Hf(x_0, y_0) > 0$, $f_{yy}(x_0, y_0) > 0$, and $f_{xy}(x_0, y_0) = 0$, then f ______________ a local maximum at (x_0, y_0).

2. Let $f(x,y) = \dfrac{x^4}{4} - 2x^2 + 3y^2$.

   (a) Find all critical points of f.

   (b) Classify the critical points using the second derivative test and write down which type of point each one is according to the test. If you cannot conclude from the test, say so.

3. Let $f(x,y,z) = x^2 + y^2 + z^2 - z^3$. Find all critical points of f.

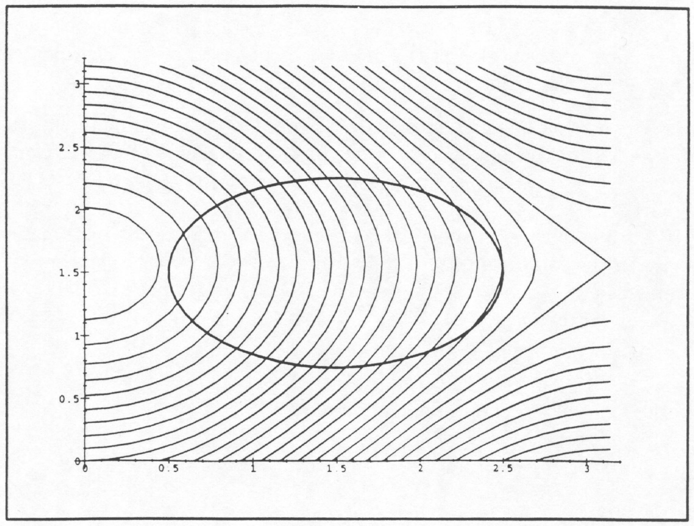
4. The function $f(x,y) = \frac{3}{2}xy - x^3 - \frac{3}{8}y^2$ has only two critical points: $(0,0)$ and $(1,2)$. DO NOT COMPUTE THEM. Instead, find the global maximum value and the global minimum value of f **in the rectangular domain R** $= \{(x,y) : 0 \le x \le 1, \ \ -2 \le y \le 0\}$.

5. (a) Using the Method of Lagrange Multipliers, set up a system of equations to find the points on the plane $x + y - z = 1$ that are closest to the origin.

   (b) Give the coordinates (x,y,z) of the points that are closest to the origin and compute the distance.

6. Use the table and the plot below to find an approximation of the maximum value of $f(x,y) = \sin(y) + \cos(x)$ on the curve $(x - \frac{3}{2})^2 + \frac{16}{9}(y - \frac{3}{2})^2 = 1$. Give the coordinates of the point and the value of the function at that point.

(x, y)	f(x,y)	(x, y)	f(x,y)
(0, 0)	1.0	(0.5,0)	0.88
(0, 0.1)	1.1	(1,0)	0.54
(0, 0.3)	1.3	(1.4,0)	0.07
(0, 0.5)	1.48	(2,0)	-0.42
(0, 0.7)	1.64	(2.5,0)	-0.80
(0, 0.9)	1.78	(3, 0)	-0.99
(0, 1.1)	1.89	(π, 0)	-1.00
(0, 1.3)	1.96		



7. Compute the double integral of $f(x, y) = 1$ over the region D enclosed by the curves $y = -\sqrt{2x+6}$, $y = \sqrt{2x+6}$ and the line $y = x - 1$

8. Find the volume of the region in the first octant bounded by the coordinate planes, the plane $y+z = 2$, and the surface $x = 4 - y^2$.

9. Evaluate the integral
$$\int_0^2 \int_{x/2}^{((x/2)+1} x^5(2y-x)e^{(2y-s)^2}\, dydx$$
using the substitution $u = x$, $v = 2y - x$.

10. Evaluate the integral of the function $f(x, y) = \cos(x^2 + y^2)$ over the disk of radius one $x^2 + y^2 \leq 1$.

11. Evaluate the integral of the function $f(x, y, z) = (x^2 + y^2 + z^2 + 3)^{-1/2}$ over the ball of radius 1 centered at the origin.

12. (a) SET UP a triple integral to compute the total mass of the solid in the first octant obtained by removing the cylinder $x^2 + y^2 = 1$ from the sphere $x^2 + y^2 + z^2 = 4$, if the density of mass is $\delta(x, y, z) = z^2 + x^2 + y^2$. DO NOT COMPUTE THE INTEGRAL.

    (b) SET UP a triple to compute the total mass of the solid that lies between the spheres $x^2+y^2+z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant, if the density of mass is $\delta(x, y, z) = x^2 + y^2 + z^2$. DO NOT COMPUTE THE INTEGRAL.