LINEAR ALGEBRA
PRACTICE PROBLEMS FOR THE FIRST EXAM

1. Fill in the blanks with ONE of the following words: ALWAYS, SOMETIMES or NEVER.

(a) If the dot product of two non-zero vectors is zero then the two vectors _____ are linearly independent.

(d) If two non-zero vectors line**(e)** Four different non-zero vectors in V^3 are _____ linearly independent.

(f) Four different non-zero vectors in V^4 are _____ linearly independent.

(g) Four different non-zero vectors in V^5 are _____ linearly independent.

(h) If A is an $n \times n$ matrix and $\det(A) \neq 0$, then the row vectors of A are _____ linearly independent.

(i) If A is an $n \times n$ matrix an eigenvalue of A. ix with two different real eigenvalues and $\text{trace}(A) = 0$, then the eigenvalues of A _____ hmetric equati3,-2,8).-x-2y+z=1.

4. Do the following three planes intersect? If so find their intersection.

a) $x - 2y + z = 1$, $-x - 2y + z = 1$, and $3x - 6y + 3z = 3$.
b) $x - 2y + z = 1$, $-x - 2y + z = 1$, and $3x - 6y + 3z = 2$.
c) $x - 2y + z = 1$, $-x - 2y + z = 1$, and $2x - 6y + 3z = 2$.

5. Solve, if possible, the following system of equations.
$_1 - 2x_2 + xx_2 + x_3 + x_4 = 0$, $4x_1 - 2x_2 + x_4 = 3$, $x_1 - x_3 + 2x_4 = -1$.

6. Given the matrix

$$A = \begin{pmatrix} 3 & 0 & 1 \\ -1 & 0 & 2 \\ 1 & 1 & 2 \end{pmatrix},$$

a) Compute A^{-1}.
b) Solve the equation

$$A \cdot X = \begin{pmatrix} 3 & 3 & 1 \\ 1 & 2 & -2 \\ 0 & 1 & 2 \end{pmatrix}.$$

c) Solve the equation

$$A^2 \cdot v = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}.$$

7. Compute the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix},$$

8. The matrices

$$A = \begin{pmatrix} a_{11} & 1 \\ -2 & a_{22} \end{pmatrix}, \quad \text{and} \quad B = \begin{pmatrix} 2 & 0 \\ 0 & 6 \end{pmatrix}$$

have the same eigenvalues. Compute a_{11} and a_{22}.

8. Let A be the matrix of problem 6. Which of the following statements, if any, are not equivalent to the others?

a) A is non-singular.
b) $\det(A) \neq 0$.
c) The column vectors of A are linearly independent.
d) The equation $A \cdot v = \text{col}(0,0,0)$ has only the trivial solution $v = \text{col}(0,0,0)$.
e) $A \cdot B = I$ for some matrix B.